Risp 8: Teacher Notes

Suggested use: to consolidate/revise Simultaneous Equations

Skills included: proof, mathematical language and argument

Suppose a student chooses the simplest possible set of six numbers from an arithmetic sequence, 1, 2, 3, 4, 5 and 6. These generate:

$$x + 2y = 3$$
$$4x + 5y = 6$$

Note that the student has created this example for themselves, which adds a real motivation to solve it. The solution is $x = -1$, $y = 2$. Teacher time at this point should be devoted to the weaker students: can you solve these equations? What happens if we interpret these as graphs? What if we make these equations even simpler? Taking -2, -1, 0, 1, 2 and 3 as your terms generates:

$$-2x - y = 0$$
$$x + 2y = 3$$

Once again we get the solution $x = -1$, $y = 2$. The truth begins to dawn that building a pair of simultaneous equations with coefficients from an arithmetic sequence in this way always gives you (-1,2) as a solution. Often the problem with improvised simultaneous equations is that they have nasty solutions. Students get bogged down with fractions, and the intended techniques get lost. Here you can have huge numbers, and stretch equation-solving powers to the limit, safe in the knowledge that the answer will be a clean (-1, 2) every time.

Meanwhile the stronger students will have been turning to algebra. (This activity really does differentiate itself nicely.)

$$ax + (a + d)y = a + 2d$$
$$(a + 3d)x + (a + 4d)y = a + 5d$$

Subtracting, we get $3dx + 3dy = 3d$, so $x + y = 1$. So $y = 1 - x$, and substituting back into the first equation gives $x = -1$, $y = 2$.

So we have that choosing coefficients in this way gives a solution of (-1, 2). Is the converse true? No, since

$$4x + 5y = 6$$
$$8x + 7y = 6$$

have the solution (-1, 2) too. A little revision gives the mini-theorem:

The first three coefficients come from an arithmetic sequence, and so do the second three,

if and only if the solution to the two equations is (-1, 2).

There are all sorts of extensions to this work. Picking coefficients from the Fibonacci sequences give the answer (1,1), for fairly clear reasons. Why not look at simultaneous equations in three variables? (If students want to do this, they will appreciate the use of a graphics calculator to do the donkey work for them.)