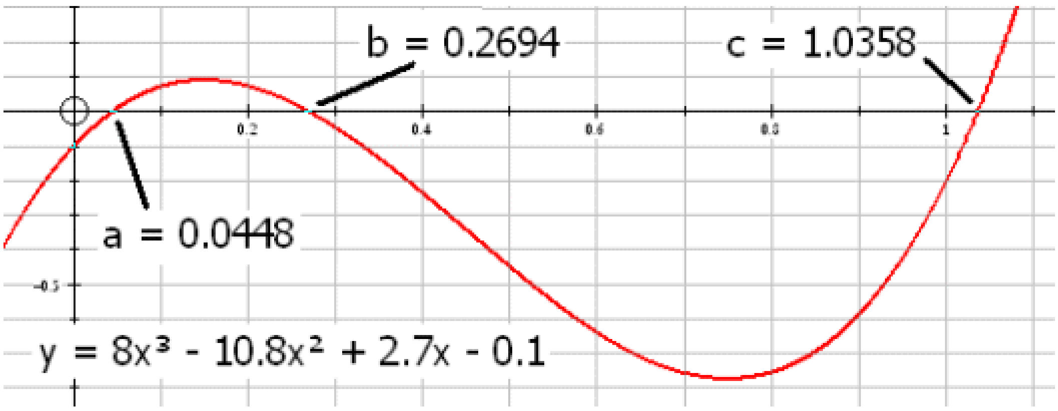
# Risp 6: Teacher notes

*Suggested use: to revise ideas of polynomials/curve-sketching*

*Syllabus areas covered:*
*Basic algebra, including expanding brackets*
*Knowing how to solve an equation graphically*
*Knowing how to sketch the graphs of polynomial functions,*
*especially in factorised form*
*Knowing how to sketch the curve of $y = f(x) + a$ given the curve $y = f(x)$*

I have changed this risp a great deal since I first added it to the site. Each time I tried the original with students, the logic of the activity felt uneasy. The task was too linear: there was not enough preliminary 'messing about'. I was being too prescriptive - "I have something in mind that I wish you to discover, and by golly, you will discover it."

This revised approach is much more rispy I think. It starts with students playing on a computer: the teacher is not the centre of attention, and the initial task is to get the screen to look a certain way, which is accessible to everyone.




Here we have an example of p, q, r, a, b, and c all being positive. So the cuboid has sides of length 0.0896 cm, 0.5388 cm, and 2.0716 cm, which gives:

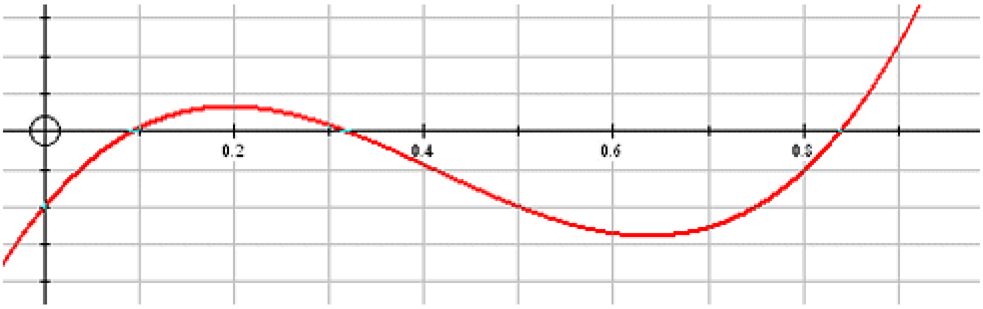
$$V = 0.1 \text{ cm}^3 \ (= r),\ S = 2.7 \text{ cm}^2 \ (= q), \text{ and } E = 10.8 \text{ cm} \ (= p).$$

Surely a mathemagical trick to impress any student, however sceptical. To prove this always works, you might invite a student to expand $(2x - i)(2x - j)(2x - k)$.
*(Note: $(2x - i)(2x - j)(2x - k) = 0$ has roots $i/2$, $j/2$ and $k/2$.)* This gives:

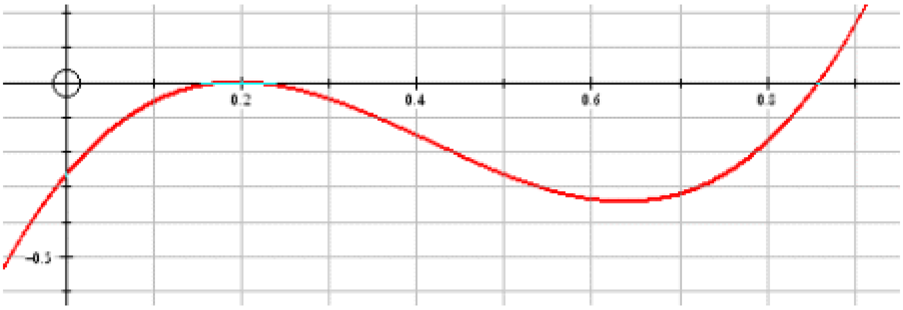
$$8x^3 - 4(i + j + k)x^2 + 2(ij + jk + ki)x - ijk$$

or $8x^3 - Ex^2 + Sx - V$, where E, S and V are the edge-length, surface area and volume of the cuboid with sides i, j and k.

So now we can turn the problem around. If E is 10 cm, and S is 3 cm$^2$, we have the graph $y = 8x^3 - 10x^2 + 3x - V$. Suppose we choose $V = 0.2$ cm$^3$ to start with:




To maximise V, we need to increase the value 0.2, which will lower the curve, until we only just have three real solutions, that is, until the curve touches the x-axis.




This gives a maximum value for V of 0.26408 cm$^3$, when the sides are 0.392 cm (twice) and 1.716 cm. V takes a minimum value of 0 cm$^3$ when the sides are 1 cm, 1.5 cm and 0 cm.

If you have 3-D Autograph available to you, you can draw $4(x + y + z) = 10$ and $2(xy + yz + zx) = 3$, and then add $xyz = k$. When is k a maximum if we still have an intersection point?

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