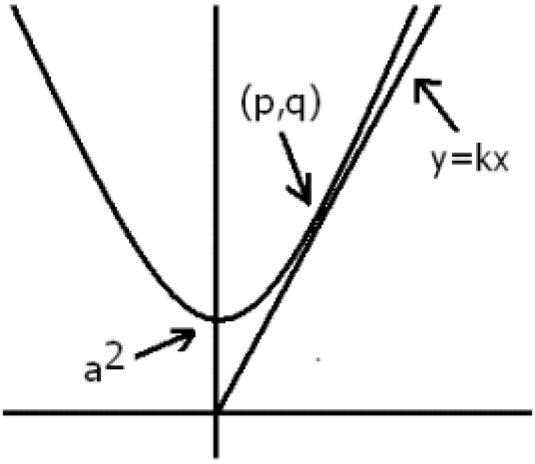
<u>Risp 5: Teacher Notes</u>

Suggested use: to revise coordinate geometry

Skills included:
co-ordinate geometry of straight lines and parabolas
simultaneous equations: solving a line and a curve
simple differentiation
equal roots for a quadratic equation
constructing a logical argument

One of the crucial things in setting a risp is to pick the right time in the year. Will your students have the maturity to tackle this now? How much prior knowledge do you want your students to have? For this risp, I chose a set of students nearly two terms through their AS year. They were comfortable with coordinate geometry, they had covered simple differentiation, and they could succeed in solving some quite hard sets of simultaneous equations. This risp turned into a chance to practice all of these skills, as well as those of sound logical reasoning.



Draw the parabola, then use the Constant Controller to adjust k so that touching takes place. Eventually a can also be entered as a constant to be varied.

How to deal with this mathematically? We can write down the following equations:

(p, q) is on the curve, which gives $q^2 = p^2 + a^2$

(p, q) is on $y = kx$ which gives $q = kp$

The gradient of the curve at p is k, which gives $k = 2p$.

Thus $q = 2p^2$, and so $p^2 = a^2$, so $p = \pm a$.

Thus $q = 2a^2$, and $k = \pm 2a$.

The other way to see this is by saying $y = kx$ and $y = x^2 + a^2$ must give equal roots when solved together.

To demonstrate this method on the second problem:

solving $y = kx$ and $y = x^2 + bx + a^2$ together,

$$kx = x^2 + bx + a^2, \text{ so } x^2 + (b - k)x + a^2 = 0$$

Equal roots mean $(b - k)^2 = 4a^2$,

so $b - k = \pm 2a$, so $k = b \pm 2a$

Everyone in the group was able to spot the first relation ($k = \pm 2a$) by experiment, and most were able to find the second ($k = b \pm 2a$) with a little help. Justifying this was a good exercise.

When I have a bad day with a risp, when students do not warm to an activity as I had hoped, when they complain that I am asking too much of them, I sometimes doubt this whole project. When that happens, I think back to trying this particular task, and I remember why I am doing this. Sitting in pairs at computers running *Autograph*, my students became engrossed as the room gradually fell silent. There was an intensity as they worked, a blessed atmosphere in which they forgot that I was in the room. For half an hour, the maths was everything. Then the chance came to put together what had been learnt and to build upon it. At the end of the lesson, one student stayed behind to try to express how good it had felt. So it wasn't just me...

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