# Risp 4: Teacher Notes

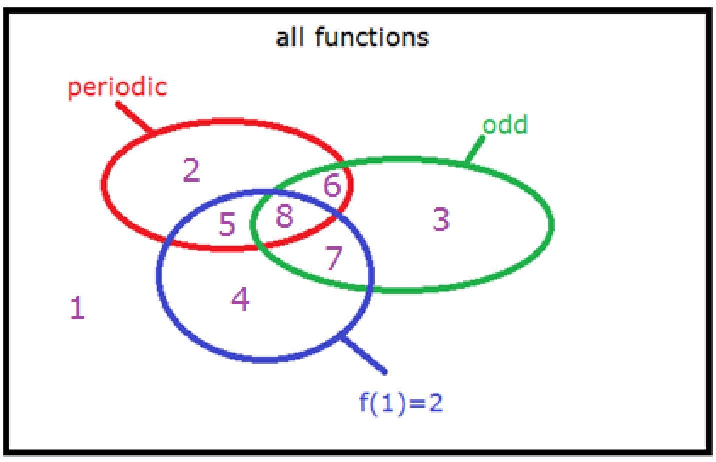
*Suggested use: to consolidate/revise functions (especially periodic functions)*

*Skills included: periodic, odd, even functions, the composition of functions*
*The transformation of graphs: deriving the graph of y = kf(x) from y = f(x)*

This activity includes several useful techniques for risp-construction. The ‘Give me an example of...and another...and another...’ idea I first met at a workshop led by Anne Watson. We were asked to think of two numbers with a difference of 9, and another pair, and another pair... What would this do to you? For me, the first example came came trivially enough, but each time I was asked for another case, I felt implicitly challenged to come up with something significantly new, some way of looking at the problem that my colleagues might not have found. We then moved on to pairs of numbers differing by π. Does the question become easier or more difficult if you change the number 9? Comparing notes afterwards, we found my examples had been of the form n and n - π, while my partner had chosen examples of type nπ and (n-1) π. So there is more than one way to skin a cat...

There is much that arises naturally from this risp: for example, does it make sense to say that if f is non-periodic that per (f) = 0? (I am grateful to Ian Short for asking: is a constant function a periodic function? If so, what would the period be?) My students happily came up with sin x, cos x and tan x as periodic functions, but I like to introduce x - [x] too, where [x] is 'the integer part of x.' This 'saw-tooth' function has period 1. Converting these into functions with period 10 is clearly a vital skill, yet some found this hard. Graphics calculators are a help with this risp, allowing students to try ideas out.

The three-property, eight-region Venn diagram is another easy way to generate richness very simply. This will work wherever you have a set of mathematical objects with (at least) three different attributes they may or may not possess. With younger children you could pick the natural numbers up to 20, and the attributes 'prime', 'even' and 'square'. Or when dealing with sequences, you could have the attributes 'oscillating', 'periodic' and 'convergent'. For our current risp, possible answers are below:




1. $x^2$
2. $\cos x$
3. $1/x$
4. $x+1$
5. $2\cos(x-1)$
6. $\sin x$
7. $2x$
8. $\dfrac{2\sin x}{\sin 1}$

Should I write per(f) or per(f(x))? There is a looseness of notation in my presentation of this risp which might make some feel nervous. If there is a choice between being rigorous in a way that obscures the meaning and unconventional in a way that enhances the meaning, I go for the latter.

Here I wanted to emphasize that while a function is usually a function of something, there are times when a function can be the something!

If you add two periodic functions, do you always get a periodic function? The perhaps surprising answer is no. Consider sin x and sin (x√2). The function sin x has period 360, while sin (x√2) has period $\frac{360}{\sqrt{2}}$. So if their sum is periodic, 360m = $\frac{360n}{\sqrt{2}}$ for some natural numbers m and n. This gives √2 = $\frac{n}{m}$, which can lead to a highly profitable discussion!

Mark Cooker from the UEA helpfully shared this problem, one that he uses with his undergraduate students: 'Show that f(x) = cos(x) + cos(x√2) is not periodic.' Mark's solution: show that although f(0) = 2, f(x) is never again equal to 2.

Yet another risp-builder can be seen in the final section: 'Are the following statements always true, sometimes true, or never true?' Suppose you are working in a topic area where there are certain classic boo-boos that every class will make at some stage. Why not put these errors centre-stage alongside some rather truer statements, and ask students to sift them into the above three categories?

For the risp we have here:

1. per(sin(x)cos(x)) = 180, not 360 × 360. However, per((x - [x]) cos(x)) = 360 = 1× 360. So 1. is sometimes true.

2. Generally, per(kf) = per (f). So as k cannot be 1, 2. is never true.

3. Given f with per(f) = a, and g with per(g) = b.

If per (f + g) = a + b, then a + b = ma, and a + b = nb,
for some natural numbers m and n. [This needs a little justification!]
So m = 1 + $\frac{b}{a}$, and thus $\frac{b}{a}$ is a natural number.
But n = 1 + $\frac{a}{b}$, so $\frac{a}{b}$ is a natural number also.
So a = b, but then per(f + g) = a, and not 2a. Contradiction!

Is this a bit of a surprise? per(sin (x) + cos (x)) = 360 < 720, yet per (sin (5x) + cos (12x)) = 360 > 72 + 30. Isn't there somewhere in the middle where equality holds?

A further extension could look at the composition of functions.

The questions above arise naturally, yet they do not have solutions that many A Level students can be expected to find. Does this make this risp too dispiriting to set? Are the harder questions included more for the pleasure of the teacher than to address the needs of the students? Could this be an example of 'teacher lust' in action? Every teacher should ask themselves these things before embarking on any risp.

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