Risp 39: Polynomial Equations with Unit Coefficients

We are going to try to find the roots of the equation:

$$x^n = x^{n-1} + x^{n-2} + x^{n-3} \ldots + 1$$

where n is a positive integer.

For example:

$$x^4 = x^3 + x^2 + x + 1$$
$$x^7 = x^6 + x^5 + x^4 \ldots + 1$$
$$x^{12} = x^{11} + x^{10} + x^9 \ldots + 1$$

How many roots would you guess these equations have?

Think about $y = x^n - x^{n-1} - x^{n-2} - x^{n-3} \ldots - 1$.
Why would drawing this be a help in finding these roots?

Use a graphing package to draw this kind of curve
for a range of values of n.

Are there any points common to all the curves?
Can you find the roots approximately?
What if we wish to find these roots very accurately?
What happens to the roots as n tends to infinity?

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