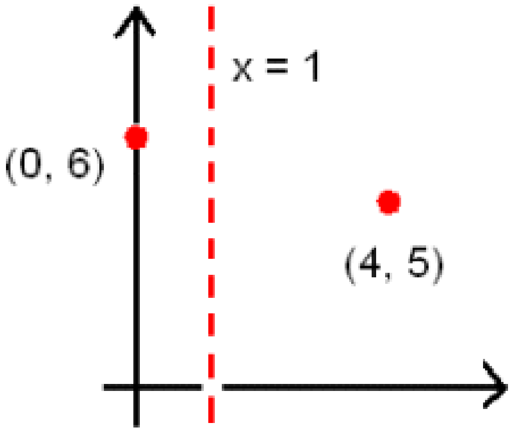
Risp 37: Teacher Notes

One of the best uses for a risp is to revisit material in a fresh way with it, some time after the basics have been learnt. A risp activity is synoptic, building and strengthening links between the different parts of a student's existing understanding, and can helpfully reveal parts of a student's mental socks where there are holes to be darned.

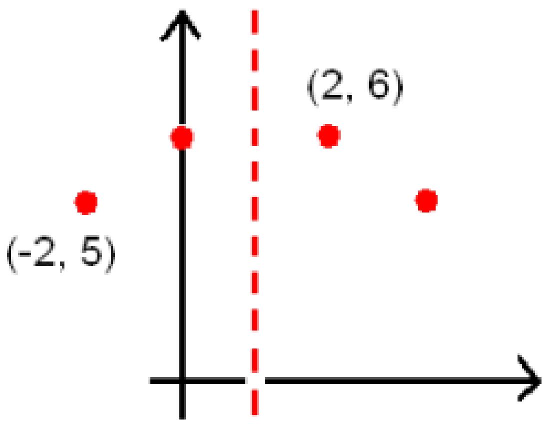
To start with, my group scratched away at this risp without making huge progress. They could immediately see why clues 3 and 4 were incompatible, but they found it hard to come up with the winning combinations.

Does this explanation work? Two unknowns need two equations for a solution, three need three and so on. So we need three clues to find a, b and c. However, some of the clues we are given here are 'single' clues (1, 2 and 4) while 3 is a 'double' clue (we might see 3 as a Goes-Through-A-Point clue + a Line-Of-Symmetry-Is clue.) So that gives us as possible working combinations [1, 2, 4], [1, 3] and [2, 3]. ([1, 2, 3] might be discussed, but that is in effect four clues that are contradictory.)

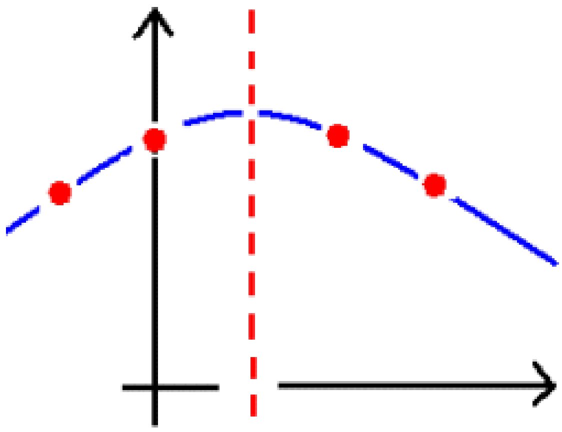
It is amazing how many students try to embark on this without a diagram. Why is such a helpful thing ignored? Let us start with [1, 2, 4]. Sketching what we know gives this:



The temptation is to insist that the parabola points downwards, in other words, to assume that a is positive. I heard several students claim that 1, 2 and 4 together was 'impossible'. But eventually we agreed that symmetry means we have two more points on the curve:



Now a parabola seems perfectly possible, with a being negative.



The equation of the parabola is clearly $y = ax^2 + bx + 6$. Putting (4, 5) and (2, 6) into this gives two equations for a and b, that yield $a = -\frac{1}{8}$ and $b = \frac{1}{4}$.

$$\text{So } y = -\frac{1}{8}x^2 + \frac{1}{4}x + 6.$$

Moving on to clue combination [1, 3]; we know that $y = ax^2 + bx + 6$ again. We also know that (2, 3) and (4, 6) lie on the curve, so we use the method above to get:

$$y = \frac{3}{4}x^2 - 3x + 6.$$

However, there is alternatively the chance at this point to revise that tricky technique, 'completing the square. If (2, 3) is a turning point, then the equation of the parabola must be:

$$y = a(x - 2)^2 + 3.$$

Now putting (0, 6) into this gives the required value for a.

The clue combination [2, 3] follows similarly; we know $y = a(x - 2)^2 + 3$, so put (4, 5) into this, giving:

$$y = \frac{1}{2}x^2 - 2x + 5.$$

Another approach would be to recall that the line of symmetry of a parabola has the equation $x = -\frac{b}{2a}$ (This was not generally well remembered!)

I would guess that if I had presented my students with this risp just as they were meeting quadratic functions for the first time, it would have scared off the less confident. By the start of their AS revision work, however, most had developed a mathematical maturity that enabled them to get plenty out of the activity. Lots of interest was evident, and possible extensions to cubics and quartics were discussed. The 'Here are some clues – which combinations work?' model for a mathematical task can be adapted for almost any area you care to name.