Risp 35: Teacher Notes

Simple to state, the starter problem involving the three great constants Φ, π and e here is one for which our previous mathematical education may have left us disturbingly unprepared. Estimating the sizes of the six numbers proves remarkably difficult, yet we feel that it is a fair question, one that we should be able to make a reasonable fist of. It turns out that to the nearest whole number:

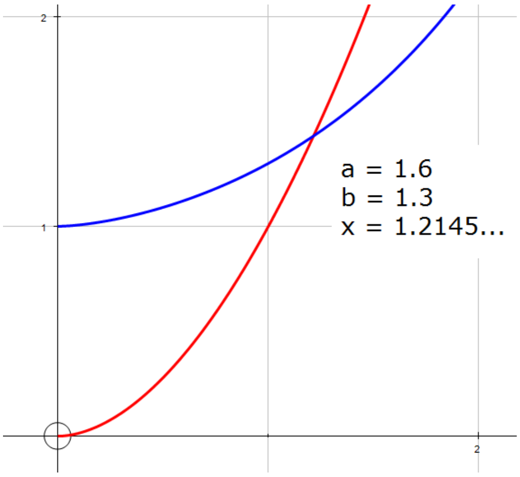
$$\pi^{(\phi^{e})} = 69, \quad e^{(\phi^{\pi})} = 93, \quad \pi^{(e^{\phi})} = 321$$

$$e^{(\pi^{\phi})} = 586, \quad \phi^{(\pi^{e})} = 49\,395$$

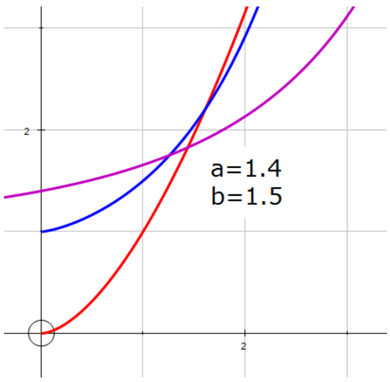
$$\phi^{(e^{\pi})} = 68\,567$$

So k is 1000 (and how many people guessed that?)

A graphing package is once again our friend for the later parts of the risp. We can plot y = x ^ (a ^ b) and y = b ^ (x ^ a) and find that there is no problem locating (x, a, b) triples where the two curves meet.

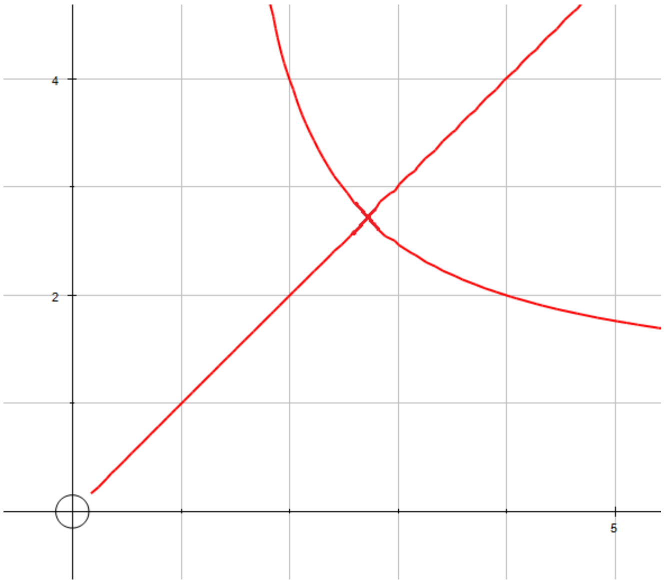


However, when we add the curve y = a ^ (b ^ x), the only time the three curves are coincident seems to be when two of the three numbers x, a and b are identical.

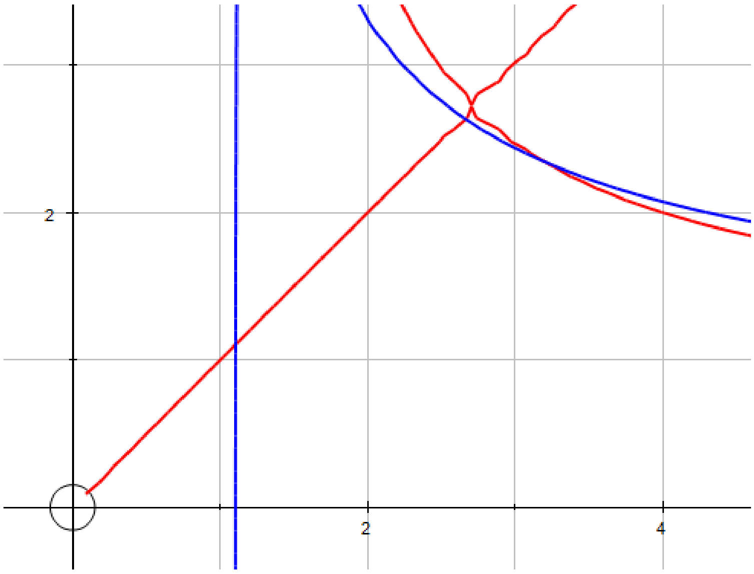


A challenge; can anyone supply a proof for this?

Moving to the final part of the risp, we can get three of these numbers to be equal. Let us aim for a ^ (x ^ y) = a ^ (y ^ x) = x ^ (a ^ y). For the first part of this we need x ^ y = y ^ x, and Autograph will draw us this curve implicitly.



If we now try to add the graph a ^ (x ^ y) = x ^ (a ^ y), we may find that our graphing package baulks at this. However, (x ^ y) ln a = (a ^ y) ln x treats our computer with more kindness.



The curves coincide at, for example, a = 1.1, x = 3.222..., y = 2.333... We can then check that 1.1^(3.222^2.333) = 1.1^(2.333^3.222) = 3.222^(1.1^2.333) = (approx) 4.309...

Note: the solution here is not $a = \frac{11}{10}$, $x = \frac{29}{9}$, $y = \frac{7}{3}$.

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