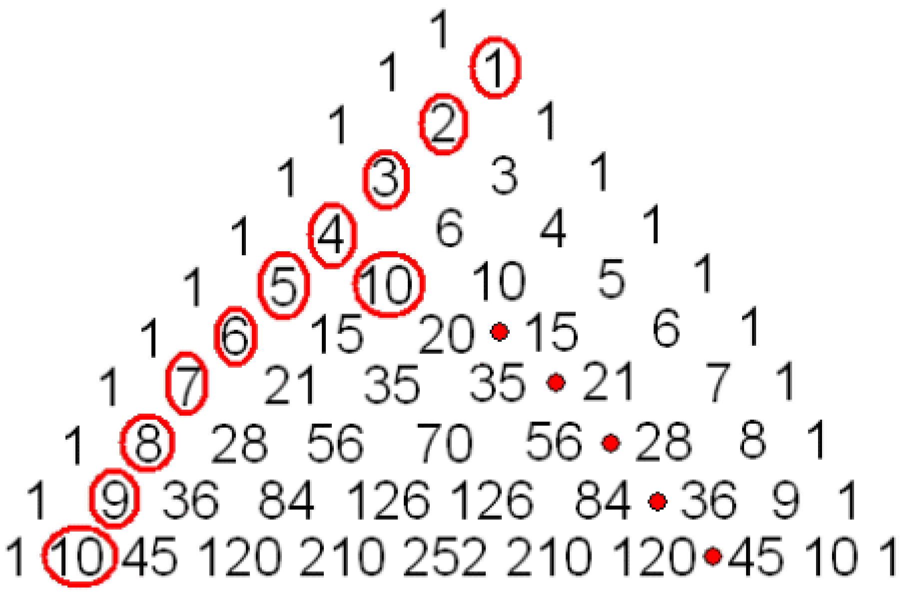
Risp 32: Teacher Notes

Suggested use: to consolidate/revise Pascal's Triangle, Binomial coefficients

One small comment in the classroom can give birth to a substantial risp. I am grateful to Colin Foster for relaying to me the remark from one of his students that begins this activity:

$$"{}^5C_2 = 10, \text{ so } {}^nC_r = nr."$$

If only things could always be so simple. Certainly there are other examples where this holds; ${}^nC_1 = n$, so we have an infinite set of examples here. It seems a good idea to draw Pascal's Triangle and to mark in with a red dot where nr overtakes ${}^nC_r$, as eventually it must.



It is a small step from here to come up with a double conjecture:

1. ${}^nC_{n-3} > n(n-3)$ and 2. ${}^nC_{n-2} < n(n-2)$ both for $n \geq 6$.

Let us take 1 first.

$${}^nC_{n-3} = \frac{n!}{(n-3)!3!} = \frac{n(n-1)(n-2)}{6}.$$

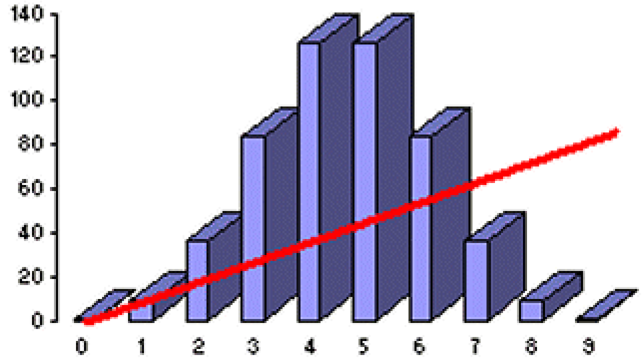
$$\frac{n(n-1)(n-2)}{6} - n(n-3) = \frac{n}{6}(n^2 - 9n + 20) = \frac{n}{6}((n-4.5)^2 - 0.25) > 0 \text{ for } n \geq 6.$$

Now taking 2:

$${}^nC_{n-2} = \frac{n(n-1)}{2} \text{ and } n(n-2) - \frac{n(n-1)}{2} = \frac{n(n-3)}{2} > 0 \text{ for } n \geq 6.$$

Risp 32: Teacher Notes (continued)

It might help to think of this graphically. Below is the example $^{9}C_{r}$.



So will nr be closer to $^{n}C_{n-3}$ or $^{n}C_{n-2}$?

$^{n}C_{n-3}$ increases as n^3 while nr and $^{n}C_{n-2}$ increase as n^2, so for $n \geq 10$, nr is closer to $^{n}C_{n-2}$.

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