Risp 31: Teacher Notes

Suggested Use: to consolidate/revise logarithms

This is a tough but rewarding risp that can be used to revise logs and their properties. The task differentiates well; there are some simple equations that all students can create and dissect, whilst it is not hard for stronger students to construct more demanding equations that need plenty of thought. Below are nine possible equations that may come up.

$$\log_a a = \log_b b$$

This is always true.

$$\log_a a = 0$$

This is never true.

$$\log_a b = 0$$

This is sometimes true (if b = 1).

$$\log_a b = \log_b a$$

Say $\log_a b = \log_b a = k$. Then $a^k = b$ and $b^k = a$, so $a^{k^2} = a$, so $k^2 = 1$. Thus $k = 1$ or -1, so this equation is sometimes true. There is the obvious solution here of $a = b$, but additionally $\log_a(1/a) = \log_{(1/a)}a$, which is worth spotting.

$$\log_a \log_b b = 0$$

There is one way to make sense of this, $\log_a 1 = 0$, which is always true.

$$\log_a \log_b \log_c c = 0$$

Again, the only way of reading this sensibly gives 'never true'.

$$\log_a b + \log_b a = \log_c c$$

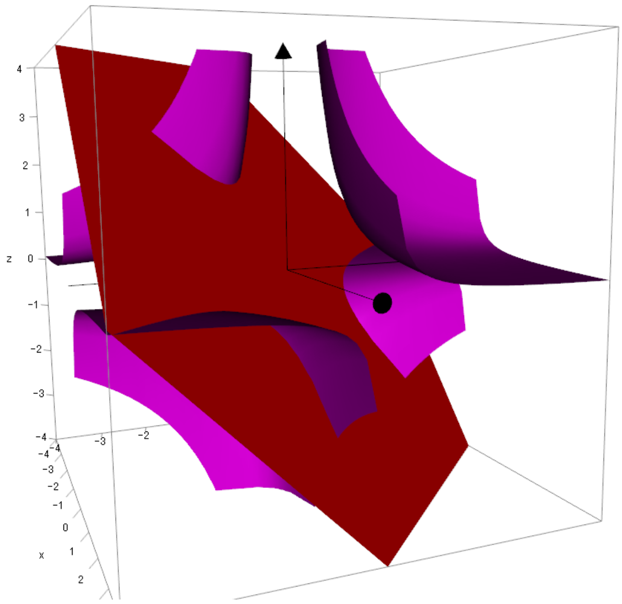
We could write this as $x + y = 1$, where $a^x = b$ and $b^y = a$, so $a^{xy} = a$. So we need to solve $x + y = 1$ with $xy = 1$, and these equations have no common solution (think about their graphs.) So this is never true.

$$\log_a b \log_b a = \log_c c$$

Using the same technique as above, this is always true. [With thanks to Bernard Murphy.]

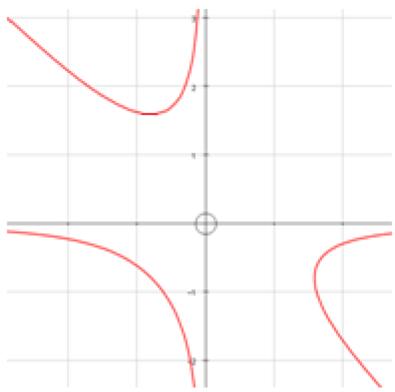
$\log_a b + \log_b c + \log_c a = 0$

This could be written as x + y + z = 0 (surface 1), with $a^x = b$, $b^y = c$ and $c^z = a$, which gives $a^{xyz} = a$, or xyz = 1 (surface 2). It is helpful to draw this pair of surfaces in 3-D Autograph, and you can see the intersection points.



Note that while a, b and c have to be positive, x, y and z do not.

Alternatively we need $x + y + \frac{1}{xy} = 0$*, an elliptic curve which provides plenty of possibilities, for example,* $x = 2$, $y = -1 + 1/\sqrt{2}$, $z = -1 - \frac{1}{\sqrt{2}}$.



So this last equation is sometimes true.

www.risps.co.uk