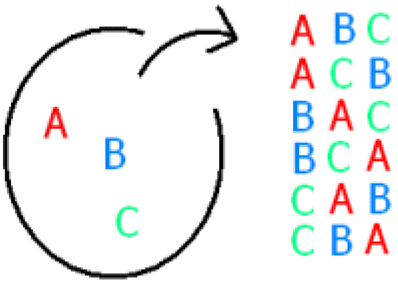
<u>Risp 3: Teacher Notes</u>

Suggested Use: to introduce/consolidate/revise simple expanding/factorising

Skills included:
general use of algebra, including expanding brackets, and factorising a quadratic
proof and mathematical argument

Everyone who tries to write their own materials for their classroom develops a style. I very often use this idea of choosing a number of objects and then mixing them up in all possible ways within an expression or equation.



This generates a family of problems, and if you can find a property that links the results, you have a chance to set students plenty of practice at a particular skill whilst asking them to discover this property, or 'mini-theorem'.

For me this is so much better than getting your students to plough through exercise after exercise with little over-arching motivation. I would say that it is the difference between working out alone at the gym and taking part in some well-matched team game. (Although I concede that some people <u>do</u> like to pump iron at the gym!)

Overall, this worked well as a first-risp-of-the-year. The reality of the lesson was that students drifted in over a period of half an hour as they were gradually given their timetables. Using a risp is helpful in this situation; incoming students join a working atmosphere, while there is no danger of the early students running out of work. This risp required no introduction: everybody recognised that they already had most of the skills required to get going.

So what would a typical attempt at this look like? Suppose I choose the numbers 2, 3 and 4.

$$(x + 2)(3x + 4) = 3x^2 + 10x + 8$$
$$(x + 2)(4x + 3) = 4x^2 + 11x + 6$$
$$(x + 3)(2x + 4) = 2x^2 + 10x + 12$$
$$(x + 3)(4x + 2) = 4x^2 + 14x + 6$$
$$(x + 4)(3x + 2) = 3x^2 + 14x + 8$$
$$(x + 4)(2x + 3) = 2x^2 + 11x + 12$$

Adding the right hand sides gives $18x^2 + 70x + 52$,
which is $2(9x + 26)(x + 1)$.

It turns out that the right hand side will always factorise with (x + 1) as a factor, a fact that students can discover by comparing notes with their partners.

There are a number of helpful points for the teacher here:

1. the exercise is self-checking. If a student ends up with a quadratic expression that does not factorise with (x + 1) as a factor, they must have made an error, most likely in their expanding.

2. if a student wants help with factorising, the teacher is aided by the fact that one of the factors is fixed.

3. the teacher can use the Factor Theorem to detect a student mistake quickly. If presented with $18x^2 + 70x + 48$, then f(-1) is not zero, so there has been a mistake.

For the general proof, algebra is needed.

$$(x + a)(bx + c) = bx^2 + (ab + c)x + ac$$
$$(x + a)(cx + b) = cx^2 + (ac + b)x + ab$$
$$(x + b)(ax + c) = ax^2 + (ab + c)x + bc$$
$$(x + b)(cx + a) = cx^2 + (bc + a)x + ab$$
$$(x + c)(ax + b) = ax^2 + (ac + b)x + bc$$
$$(x + c)(bx + a) = bx^2 + (bc + a)x + ac$$

and so the total is $2(a + b + c)x^2 + 2(ab + bc + ca + a + b + c)x + 2(ab + ac + bc)$

which factorises to $2((a + b + c)x + (ab + ac + bc))(x + 1)$

As theorems go, simple fare. But I think my students experienced this risp as a kind of 'magic trick'. There was definite surprise amongst my class to find that everyone had (x+1) as a factor. I could feel curiosity building, something that makes A Level maths come alive.

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