Risp 28: Teacher Notes

Suggested use: to consolidate/revise differential equations
(variables separable only)

Here is a risp with a slightly topical flavour. As I type, the UK and the other nations of the world are bracing themselves to withstand avian flu. Although there is no real threat to humans currently, the danger is that the virus will mutate into a form that can be spread from human to human. Journalists have rekindled memories of the horrendous flu epidemic at the end of the First World War, and we are told that we are due for something similar again. It seems timely then to tackle epidemiology in a simple way within the A Level mathematics classroom.

Do I have a sadistic streak? Maybe sometimes a little hard labour does no harm in a mathematics classroom. In particular, I think there is no substitute in this exercise for actually rolling a dice 100 times and recording the results (this takes 10 minutes at most.) One could flick on an Excel program and ask it to perform this labour in a second, but would this really be appreciated by a group who had not experienced the harder route first? It makes the ten years in the question feel more like ten years. In fact, the rolling can induce a sense of curiosity and purpose into the lesson. The Excel program will come in handy eventually when you start to vary c and r. (It is hoped to add such a program to this site before too long.)

Differential equations do lie within the Core for A2, but only in their simplest form. 'Variables separable' is the only type of DE that our C4 module contains, and fortunately, the DE generated by this scenario lies within this category.

A little reflection shows that:

$$\frac{dx}{dt} = c(1 - x) - rx.$$

If $c = \frac{1}{3}$ and $r = \frac{1}{6}$ as in our starting example:

$$\frac{dx}{dt} = \frac{1}{3}(1 - x) - \frac{x}{6} = \frac{1}{3} - \frac{x}{2}$$

Separating the variables, $\frac{6}{2-3x}dx = dt \Rightarrow -2\ln(2 - 3x) = t + a$

$$\Rightarrow x = \frac{2}{3} - be^{-t/2}.$$

When $t = 0$, $x = 0.2$, so $b = \frac{7}{15}$, and we end up with:

$$x = \frac{2}{3} - \frac{7}{15}e^{-t/2}.$$

As t tends to infinity, x tends towards $\frac{2}{3}$. Hopefully this will fit with your data.

Starting in general from $\frac{dx}{dt}$ = c(1 – x) – rx,

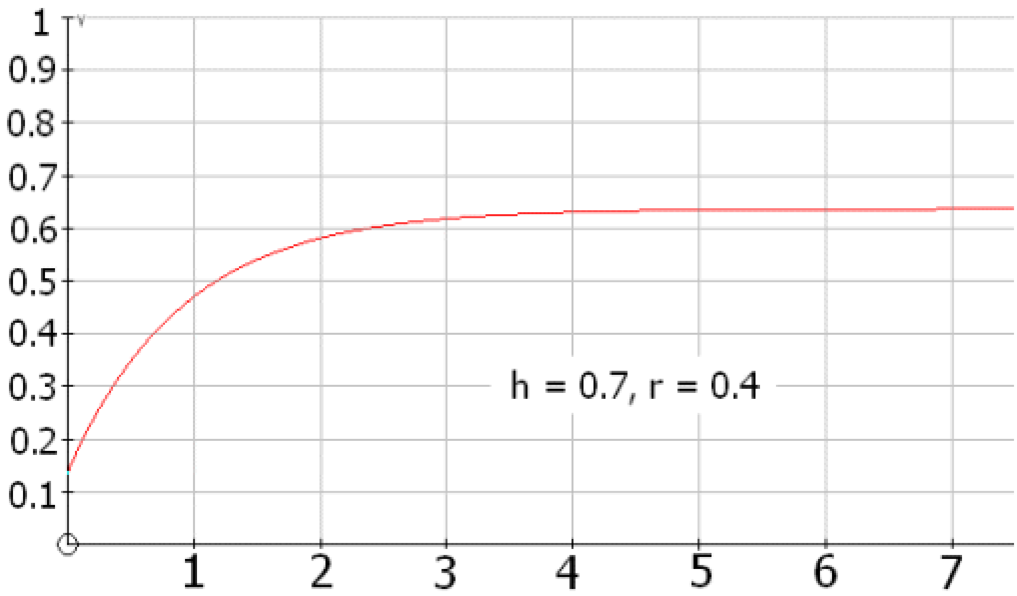
$$\frac{dx}{dt} = c - (r + c)x.$$

Separating the variables, $\frac{1}{c-(r+c)x}$ dx = dt

$$\Rightarrow \frac{-1}{r+c} \ln (c - (r + c)x) = t + a$$

$$\Rightarrow x = \frac{c}{r+c} - be^{-(r + c)t}.$$

This can be entered into Autograph, and the constants can be varied.



As t tends to infinity, x tends to $\frac{c}{r+c}$; if c is big in relation to r, $\frac{c}{r+c}$ will be close to 1.

If c = r, then x tends to $\frac{1}{2}$; if r = kc, then x tends to $\frac{1}{k+1}$.

I would not jump immediately to the general case with my students here. Indeed, this is quite a tough risp to use as an introduction to Des - it might best be used as the start of the concluding lesson on the subject.

www.risps.co.uk