Risp 27: Teacher Notes

Suggested Use: to introduce the idea of parametric equations

The first picture that we encounter on entering the curve into Autograph is this (it is a good idea to set the t-step to 0.01 for clarity):

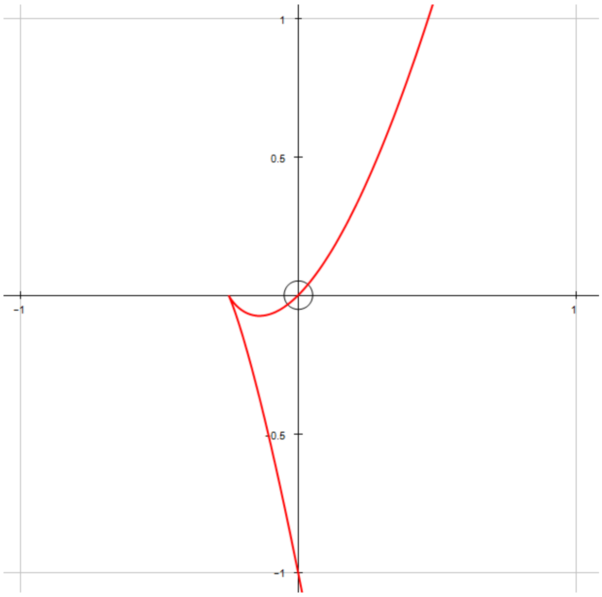

Figure 1

As we vary a and b, we may notice that the curve always goes through (0,0), that the curious spike feature (is it was up to me, I would call this curve 'The Tick') is always the most westerly point on the curve, and there seems to just one local maximum (or minimum) point on the curve besides our cusp.

When does the tick happen? If it does occur at the minimum value for x, then given x = t^2 + t,
x = $(t + 0.5)^2$ - 0.25, so the tick always lies on x = -0.25 when t = -0.5.

We have $\frac{dy}{dx} = \frac{dy}{dt}\Big/\frac{dx}{dt} = \frac{12at^2+(6a+2b)t+b}{2t+1} = \frac{(6at+b)(2t+1)}{2t+1}$.

If t is not -0.5, we can cancel straightforwardly here to find the stationary point at t = $\frac{-b}{6a}$. If t = -0.5, well, we can see strange things (like a tick) might happen here!

Finding $\frac{d^2y}{dx^2}$ yields $\frac{6a}{2t+1}$, and again, t = -0.5 leads to a need for further investigation; Figure 1 shows the tick in that case to be a local maximum. The other stationary point will be straightforwardly a maximum or minimum according to the sign of $\frac{d^2y}{dx^2}$.

Allowing a wider range of coefficients does lead to a loop on occasions; for example, here we have $x = t^2 + t + 1$, $y = t^3 + 2t^2$.

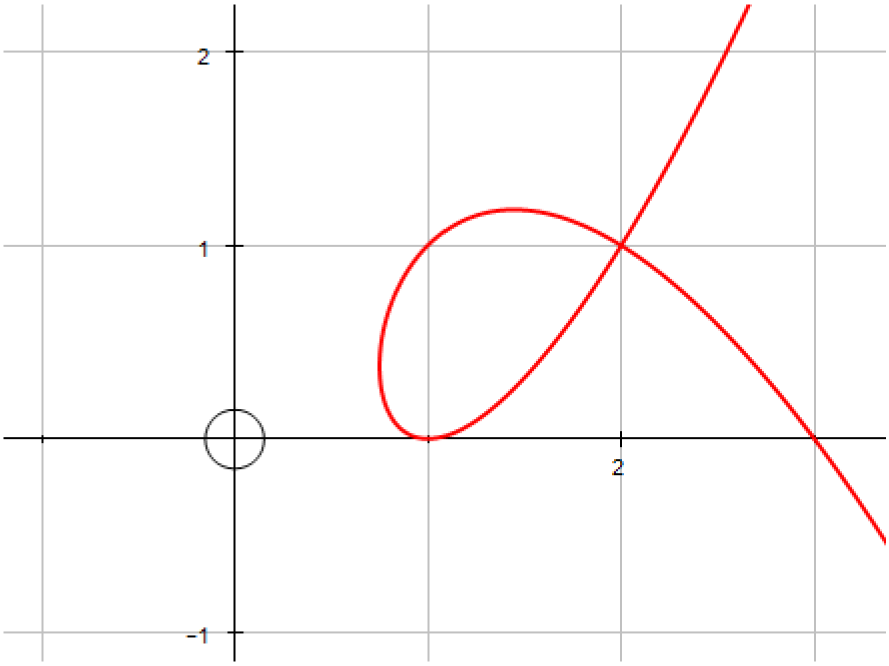

Figure 2

Finding the stationary points is good practice in parametric differentiation:
$\frac{dy}{dx} = \frac{t(3t+4)}{2t+1}$, and we can find the two stationary points clearly visible in Figure 2.

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