<u>Risp 21: Teacher Notes</u>

Suggested Use: To introduce/consolidate/revise ideas about anything!

The human race often seems to be good at doing things, but less good at undoing them. We can do nuclear power, but can we undo a nuclear power station? This may well prove to be a lot harder and more costly than we anticipate. We can pump carbon dioxide into the atmosphere, no problem, but reducing the level of carbon in the air? And we seem quite good at making other living things extinct, but bringing a dodo back from the dead so far eludes us.

So it is within our classroom. Given a rule to work with, a student may be able to follow it successfully, but as soon as some reversal is needed, some mirroring of the rule that requires understanding, they feel confused. If my students can successfully do <u>and</u> undo, then I can be confident that they feel significantly at home with the material.

There is no better way to present ideas of doing and undoing than arithmogons. They have been around a long time: 1975 is the first reference I have for them, but I daresay they have been around longer than that. One of their many marvellous aspects is that they require few words. (I have tried to manage with none at all in presenting this risp!) John Mason and Sue Johnston-Wilder (see Risp Books) give a long and fascinating study of how simple arithmogons can be used in a variety of ways.

Figure 1 is the simplest possible example. Add two circles and put the answer in the box between them. The 'undoing' version is harder, however. If we call the unknowns a, b and c, a + b = 7, b + c = 12, c + a = 9. Simultaneous equations (will your students have met three equations in three unknowns before?) giving a = 2, b = 5, c = 7. Can we find a solution whatever the numbers in the boxes are, and will there always be just one solution?

The multiplication example is a touch more advanced. Here ab = 7, bc = 12, ca = 9, giving
$a^2 = \frac{21}{4}$. So two solutions this time, $a = \pm\frac{\sqrt{21}}{2}$, $b = \pm\frac{2\sqrt{21}}{3}$, $c = \pm\frac{6\sqrt{21}}{7}$. In my experience this is a really good exercise for A Level students early in their course. It is an excellent chance to run over the theory of surds, for example that $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, rationalising fractions with a surd in the denominator and so on.

Figures 5 and 6 are directed arithmogons, for the case when the 'doing' is not commutative. Figure 6 in particular raises some interesting questions. It looks harmless enough, but how many solutions does it have? The answer is none, so what is the condition on a, b and c for there to be solutions?

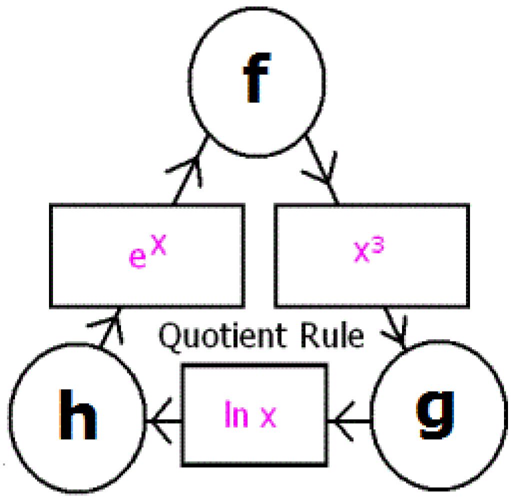
Figures 7 and 8 show how an arithmogon can be extended into any topic you care to name. Rather than set exercises from the book, these figures provide practice with a purpose. My students seem to find the sense of completeness that arises from filling the boxes and circles an aid to learning in itself.

Figures 9 and 10 move into partial fractions, a classic doing/undoing scenario. In Figure 10, it is remarkable how much information it is possible to rub out for the problem still to remain viable, providing more useful practice with simultaneous equations. The solutions here are $\frac{4}{x-3}$, $\frac{2}{x+1}$ and $\frac{6}{x+2}$.

Figures 11 - 16 address differentiation and the Product/Quotient Rules. Should you need the solutions, Figures 12 and 14 have e^{-x}, x^3 and ln(x) as answers. Figure 15 is fairly possible, I think, but Figure 16... if anyone out there has a solution, I will publish it happily!

Postscript

George Wellen of Bradfield College wrote with an answer to my challenge in the last paragraph.



We have $\frac{d\left(\frac{f}{g}\right)}{dx} = x^3$, $\frac{d\left(\frac{g}{h}\right)}{dx} = \ln(x)$, and $\frac{d\left(\frac{h}{f}\right)}{dx} = e^x$.

Integrating, we have $\frac{f}{g} = \frac{x^4}{4} + p$, $\frac{g}{h} = x\ln(x) - x + q$, $\frac{h}{f} = e^x + r$

for some constants p, q and r. Multiplying these together gives:

$$\left(\frac{x^4}{4} + p\right)(x\ln(x) - x + q)(e^x + r) = 1.$$

Allowing x to tend to infinity now shows that this cannot be for any values of p, q, and r. A sweet proof of the impossibility of Figure 16 - thank you, George.

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