Risp 2: Teacher Notes

Suggested Use: to consolidate/revise sequences/series

Skills included:
defining a sequence: the nth term in terms of n, and in terms of previous terms
identifying convergent, divergent, increasing,
decreasing, oscillating and periodic sequences

My students looked at these tiles and wondered what to do. They began to place them together in different ways, some defining legal sequences, and some not. Words like 'divergent', 'convergent', 'bounded' and 'periodic' came into play, and during lively discussion students were able to say, "I don't know what 'divergent' means," without being made to feel self-conscious. My class began to stick down tiles onto a poster to summarise our work. There was plenty of debate about what counted as convergent or divergent. Students asked if they could use spare tiles from other groups in their formulae, which I allowed. This makes $\frac{n-1}{n}$ possible as a sequence.

I was able to draw up a table of sequences we had found:

	Divergent	Convergent	Periodic	Other
Always increasing				
Always decreasing				
Oscillating				
'Flat'				

I threw out this challenge: "Can we fill the gaps using these tiles only?"

	Divergent	Convergent	Periodic	Other
Always increasing	n^n	(n-1)/n	Not possible	Not possible
Always decreasing	-n	1/n	Not possible	Not possible
Oscillating	$(-n)^n$	$(-1)^n/n$	$(-1)^n$	?
'Flat'	Not possible	un=un-1	Not possible	Not possible

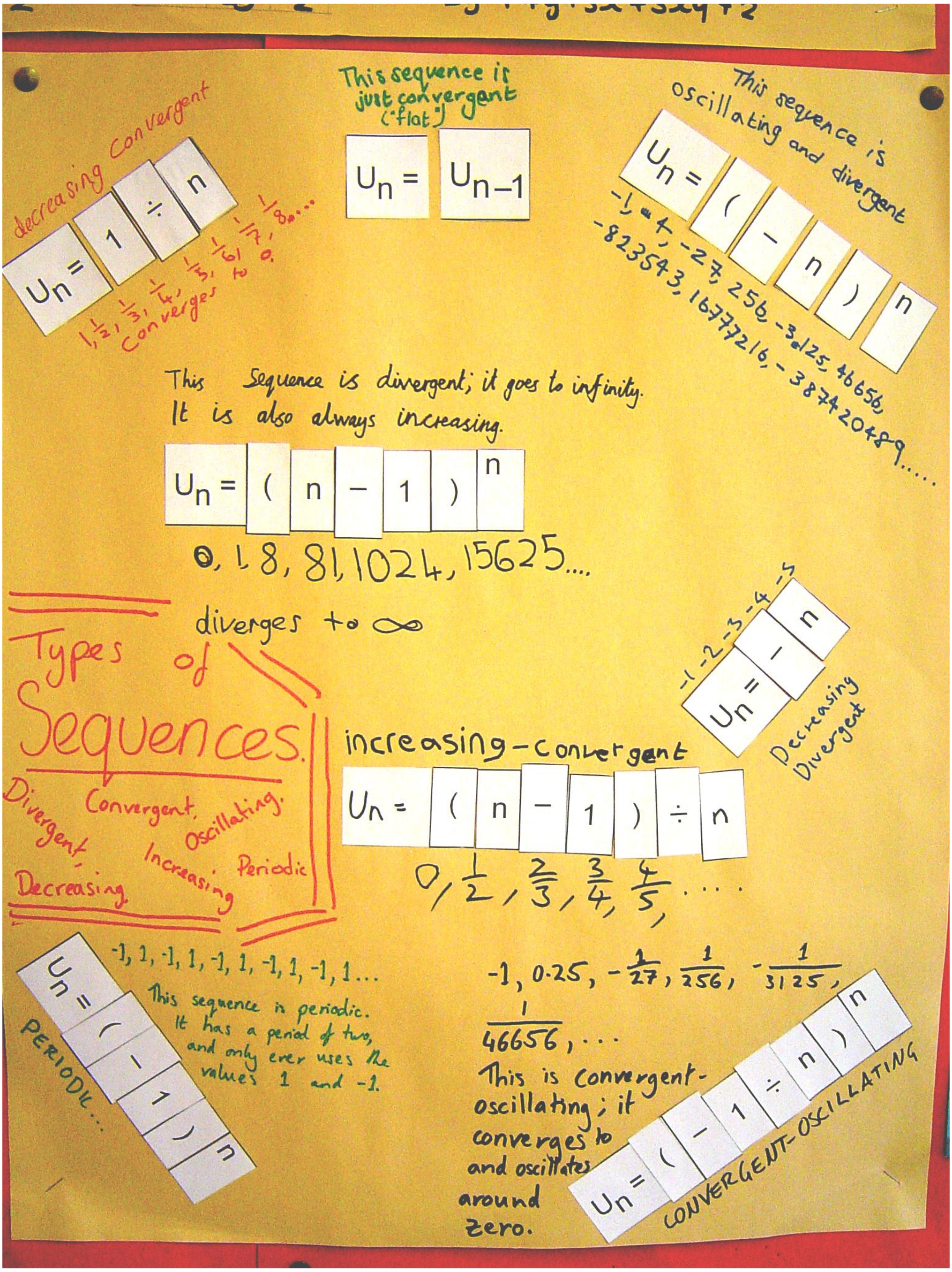
Can we have an always-increasing yet bounded sequence that is not convergent? This activity raises some deep questions. There are lots of extensions. What tile would I need to add to fill the gap marked ? above? How about a 'last digit of' tile? Would 'last digit of n' fill the gap? How about 'last digit of n^n'?

Adding in the u_{n-1} and u_{n-2} tiles creates some interesting sequences.

$u_n = \frac{1}{u_{n-1}u_{n-2}}$ is periodic, period 3, and $u_n = \frac{u_{n-2}}{u_{n-1}}$ is periodic, period 6.

The ? gap can be filled by u_n = nth digit after the decimal point in π.

Our poster looked like this:



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