Risp 19: Teacher Notes

Suggested Use: to consolidate/revise the Binomial Theorem (for n rational/negative)

Here once again, the "How many ways can we mix three numbers up?" technique is employed. I am finding that my students are beginning to recognise this kind of exercise now, and explanations are no longer needed. They worked with purpose on this risp, aided by the fact that it has a direction of its own. The result it heads towards is not deep (at least I don't think it is!), but it is good that there is a result, albeit modest, to head towards. There is a thought I like from Gary Flewelling and William Higginson (see Risp Books):

A conventional exercise asks you to write a sentence,
while a rich learning task invites you to write a story.

This risp tells a short story quietly and without histrionics (you could complete this in ten minutes on a good day.)

Another aspect to this task that I like: there are moments of rest built into it. $(2 - x)^1$ can be expanded into a + bx with ease, so all of a sudden being asked to carry out six binomial expansions doesn't look too bad. There is an element of ebb and flow. Everybody can do some of this activity with success.

Suppose I choose 31 (another nice facet to this activity is that big numbers make the calculations more impressive without complicating them too much.) Expanding as far as x in each case:

$$(1 + 31x)^{-1} = 1 - 31x + \ldots$$

$$(31 + x)^{-1} = \frac{1}{31}\left(1 - \frac{x}{31} + \ldots\right) = \frac{1}{31} - \frac{x}{312} + \ldots$$

$$(-1 + 31x)^{1} = -1 + 31x$$

$$(31 - x)^{1} = 31 - x$$

$$(1 - x)^{1/31} = 1 - \frac{1}{31}x + \ldots$$

$$(-1 + x)^{1/31} = -1(1 - x)\ \frac{1}{31} = -1 + \frac{1}{31}x + \ldots$$

These will all be valid at the same time if $|x| < \frac{1}{31}$.

Adding these gives:

$$A + Bx = (31 + \frac{1}{31}) + (-1 - \frac{1}{312})x$$

$$\Rightarrow \frac{A}{B} + n = \frac{31 + \frac{1}{31}}{-1 - \frac{1}{312}} + 31 = 0.$$

Running through this with a general odd number n rather than 31 shows that we will always get 0. Thus this risp has the self-checking properties that we have already noticed in others.

Some might say, expanding as far as the term in x is not a real test of whether the Binomial Theorem has been fully understood. I would agree, but this risp is at least a start in that direction. The way the expansion is simplified here may make the activity more accessible for weaker students, and this risp certainly tests the technique for expanding $(a + bx)^c$. Expanding further could make a sensible extension for stronger students. If the expansions add to $A + Bx + Cx^2 + ...$, then $A^3B^2C + B^5 = A^5$. If you can find a nicer relation between later coefficients, please let me know!

I am grateful to Sue Cubbon for sending in the following comment:

The group had commented in passing that they thought they had forgotten what 'Binomial' was about. I was delighted with the consolidation the risp offered, and their excited response: they were determined to prove the result and were very pleased with themselves when some could.

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