Risp 16: Teacher Notes

Suggested use: to consolidate/revise the Quotient Rule

This is a good example of how a task can be enriched. I wanted to organise some practice for my students on differentiation using the Quotient Rule. [Note: this risp assumes that students know how to differentiate e^x.] One way would be to simply set some exercises from the book, and there are occasions when it would be foolish to do anything else. But given that I had a little more time available, might there be a more oblique way to achieve my goal, by finding an activity that asked for use of the Quotient Rule along the way to examining some wider question?

It seemed natural to me to ask the class to pick two functions of x, called u and v, and then get them to differentiate f(x) = u/v. But if we call g(x) = v/u, we can get two uses of the Quotient Rule for the price of one.

$$f'(x) = \frac{vu' - uv'}{v^2}, \text{ and } g'(x) = \frac{uv' - vu'}{u^2}.$$

Can we combine these in some sensible way, so that the result has some interesting property? Multiplying gives:

$$f'(x)g'(x) = \frac{vu' - uv'}{v^2} \frac{uv' - vu'}{u^2} = -\frac{(uv' - vu')^2}{v^2 u^2}.$$

It is clear that whatever u and v are, this will never be positive. (Dividing f '(x) and g '(x) gives the same result.)

Now place this quality, never-being-positiveness, centre stage. I asked my students to play with this notion, and to examine the algebra of NP functions; I was almost forgetting the initial motivation for the idea in doing this. Now to bring in the Quotient Rule at the end seemed natural and hopefully unforced. Students were practising a target skill within a wider context.

Dinosaur bones or pottery shards alone have little scientific value. The same can said for a student trying to make sense of concepts or procedures. Removed from their contexts, concepts and procedures lose much of their significance and meaning.

A Handbook on Rich Learning Tasks, 2001,
Gary Flewelling with William Higginson (see Risp Books)

There are lots of ways to create an NP function:

$$-|x|,\ -e^{10x-5},\ -1 + \sin(x^3-3x^2-x+4),\ \text{etc.}$$

Drawing graphs helps to give an idea of what we are aiming for.

If you add two NP functions is the result NP? For sure. But if you subtract two NP functions, there is no guaranteeing the NP-ness of the result. For example, $f(x) = -2x^2$ take away $g(x) = -x^2$ gives $-x^2$, which is NP. However, $g(x) - f(x)$ gives x^2, which is NN (never negative). Multiplying or dividing two NP functions gives an NN one.

The Venn diagram works nicely here. Is it possible to find (u, v) for the central region? If u + v is NP and so is u - v, then their sum will be NP. Thus u is NP. Since uv is NP, v must be AP. How about: $u = -2x^2$, $v = x^2$? Or even more simply, (0, 0). Answers for the other regions are:

1. (2, 1)
2. $(-1, -x^2)$
3. (x, x)
4. (-2x, x)
5. (x, -x)
6. (-2, -1)
7. $(-x^2, 2x^2)$

The bag of functions is there to stop students from picking over-simple functions for the last part, thus removing the need for the Quotient Rule.

www.risps.co.uk