Risp 14: Teacher Notes

Suggested use: to introduce/consolidate/revise types of sequence

Skills included:
Types of Sequence, convergent, divergent, oscillating, bounded, periodic

Risp 2 tackles sequences, and here is another activity that does the same. The former is a superior consolidator, while this risp tends to work better as a starter on the subject.

Your syllabus deserves a careful reading over what exactly terms like 'divergent' mean. I take 1, -2, 3, -4 , 5, -6... as being divergent and oscillating, and
3, 1, 4, 1, 5, 9... as being bounded, and oscillating, but not divergent. (How would you use the word 'chaotic' to describe a sequence?)

The syllabus we use [MEI] implicitly expects students to recognise six types of sequence:

1. convergent-monotonic
2. convergent-oscillating
3. divergent-monotonic
4. divergent-oscillating
5. periodic
6. bounded, non-periodic and non-convergent

How to introduce these? The good news is that this risp leads quite naturally to five from this list.

My students started by having a play, as with all good risps. For example, if we choose a = 0.1, and b = 0.4, then the sequence goes:

0.1, 0.5, 0.2, 0.6, 0.24, 0.64, 0.256, ...

As $n \to \infty$, the terms get closer and closer to 2/3, 4/15.

Assuming we have 'convergence' here, we are effectively solving (x + b) b = x,

so the sequence gets closer and closer to oscillating between $\dfrac{b^2}{1-b}$ and $\dfrac{b}{1-b}$.
Notice that the starting value of a is irrelevant to what finally happens in this situation.

After a couple of experiments, students will tire of the calculation aspect of this activity, and the time comes for a calculator program or Excel to remove any drudgery.

I have no wish to teach anyone how to suck eggs, but below is a short program that will work on my (Casio) graphics calculator to help this exercise along.

Risp 14: Teacher Notes (continued)

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? → A : ? → B : Lbl 1 : A + B → C : C #
BC → A : A #
Goto 1 :
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 # stands for the 'Print' command.

So what happens as we choose any real value for b?

$1 \le b$, $u_n \to \infty$ (divergent-monotonic)

$0 < b < 1$, $u_n \to$ two 'limits', both positive (bounded/non-periodic/ non-convergent)

$0 = b$, $u_n \to 0$ (convergent)

$-1 < b < 0$, $u_n \to$ two 'limits', one positive, one negative (bounded/non-periodic/non-convergent)

$-1 = b$, for example, 2, 1, -1, -2, 2, 1, -1, -2, 2, 1, ...
(the sequence is periodic, period 4) *Note: the value of a does matter here!*

$b < -1$, $u_n \to +/- \infty$ (divergent-oscillating)

So the only type of sequence we cannot find through this work is the convergent-oscillating kind. A good extension question might be:

"How could you tweak the sequence-generation rules here to give us our missing sequence?"

Note that for $-1 < b < 0$ and $0 < b < 1$, we have two monotonic-convergent sequences intertwined, so to speak. Can students come up with ways of defining the nth term in terms of previous terms for each of these sequences?

Another nice question: given k, how many values of a and b give k as one of the limiting terms? If we take k = 2, then (a, b) pairs that work are:

$$(\text{any}, -1 + \sqrt{3}), (\text{any}, 2/3), (2, -1), (3, -1), (-1, -1), (-2, -1)$$

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