Risp 13: Teacher Notes

Suggested use: to consolidate/revise the integration of x^n

Skills included:
definite integrals
numerical integration including Trapezium Rule

This risp assumes that students have met the rule for finding $\int x^n dx$ for all n not equal to -1, and the idea that the area under a curve can be written as a definite integral.

Your students can construct a table of values for n with their corresponding values of k. Drawing up this table can take a lot of time, unless you use the Constant Controller to find k. The following instructions apply to Autograph:

Select the curve, right click, and you have the Enter Point on Curve option available to you. Put two points on the curve, one at x = 0 and one at x = k. Select the two points, then use the Find Area option which you will find on right-clicking. Now adjust k as required.

The final table might look like this:

n	0	0.5	1	1.5	2	2.5	3
k	1	1.310	1.414	1.443	1.442	1.430	1.414

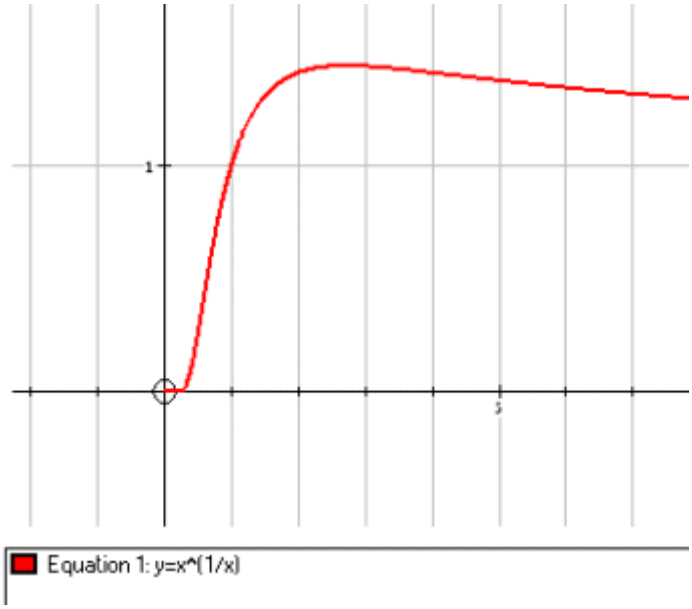
A possible range for k for n in [0, 3] would seem to be (1, 1.444...). The graph of k against n seems to have a maximum. What value of n gives that maximum?

Turning to theory:

$$\text{if } \int_0^k x^n dx = 1 \Rightarrow [\frac{x^{n+1}}{n+1}]_0^k \Rightarrow \frac{k^{n+1}}{n+1} = 1$$

$$\Rightarrow k^{n+1} = n+1 \Rightarrow k = \sqrt[n+1]{n+1}$$

We can draw the graph of $y = \sqrt[x]{x}$ using our graphing package.



The curve has a maximum when x = e, although the calculus for this would be hard for students setting out at AS Level. It is, however, a nice intuitive way to introduce the number e to students who have never come across it before; hopefully the introduction to this risp accomplishes this too.

What if n ranges across [0, ∞]?

It seems that k can be anywhere in the interval $\left(1, e^{1/e}\right]$,

with the maximum occurring when n = e - 1.

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